1 Introduction

In class (following Prialnik), we derived the Lane-Emden equation that relates density and radius for polytropic equations of state in dimensionless quantities (θ and ξ, respectively). This equation looks like

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n. \tag{1}$$

Your task is to use the Runge-Kutta method to solve this equation for a few cases of astronomical interest. Although you will use a computer to numerically solve this equation, it can be solved analytically for particular values of n. As a way to check to see if your code works, I give you now a few of those solutions:

for $n = 0$,

$$\theta(\xi) = 1 - \frac{\xi^2}{6}, \tag{2}$$

for $n = 1$,

$$\theta(\xi) = \frac{\sin\xi}{\xi}, \tag{3}$$

and for $n = 5$,

$$\theta(\xi) = \frac{1}{\sqrt{1+\xi^2/3}}. \tag{4}$$

a) First, verify that these functions actually solve the Lane-Emden Equation for the appropriate value of n, and that they have the appropriate boundary conditions.

b) Second, make sure your code can reproduce these functions to better than 1%. If your maximum deviation is more than 1%, decrease the step size and run again.

2 A White Dwarf

Compute the structure of a non-relativistic degenerate Carbon white dwarf star of mass 1.0 $M_\odot$. The equation of state for a degenerate Fermi gas is

$$P = \frac{8\pi}{15}\frac{h^2}{m_e}\left(\frac{3\rho}{8\pi\mu_e}\right)^{5/3}, \tag{5}$$

where m_e is the electron mass, $\mu_e = \rho/n_e$, and n_e is the number of electrons per unit volume. So μ_e is a mass per electron, which will depend on the composition of the star.

c) Plot density and pressure as a function of radius from the center (in SI units).

d) What is the radius of the star? What is the central pressure and density?

3 A Neutron Star

Compute the structure of a neutron star of mass 1.0 $M_\odot$. The equation of state has the same form as that of Equation 5, but the constants will be different. (Hint: use m_n instead of m_e, and $\mu_e \rightarrow m_n$, as well. This isn't because there is one neutron per electron – the derivation starts from a different assumption, so a different mass goes in that spot.)

e) Plot density and pressure as a function of radius from the center (in SI units).

d) What is the radius of the star? What is the central pressure and density?

4 Final Notes

Please work together on this, and turn in a final report with all the names of the contributers on it. Note that you can use Prialnik Eq. 5.15 and 5.20 to solve for ρ_c in terms of M, using your computer program to find ξ_1 and the derivative. From ξ_1 and ρ_c you can use 5.15 and 5.18 to get R.