Self-Directed Project 3 – Numerical Solutions of Laplaces Equation
Due 6 October, 2009, 5 pm
Worth: 15 pts toward final grade

1 Introduction

The theory behind the two dimension Cartesian relaxation method was covered in Mathematical Physics. I will be happy to clarify any questions regarding that algorithm, and a full description is given in the book by Garcia that we used for that class. For this project, you will apply this method to several situations of relevance to electrostatics. Feel free to use outside resources but be sure to acknowledge assistance where appropriate. You may program solutions for more than one of the following situations, but a maximum of 15 points is available.

2 Testing Software

Write and debug a program using relaxation in 2D (Cartesian). Make a square 100x100 array of random numbers between minus one and one. Apply the boundary conditions that one edge is at minus one, the opposite edge is at one, and the remaining two sides have linear values that connect the first two sides. Show four mesh plots that demonstrate how your program drives the interior points to the solution to Laplace's Equation. Although this section does not officially count for points, it must be completed before points can be earned for any of the following tasks.

3 Abstract Trial

(5 pts) When perfected, use the program to solve Laplace's Equation in the region with the following boundary conditions: $V(0, y) = 0$, $V(10, y) = 1$, $V(x, 0) = \sin(\pi x/20)$, and $V(x, 10) = -\sin(3\pi x/20)$.

Find V(5,6) and V(1,1) to at least 4 significant digits. Make a mesh plot of your solution.

4 Charged, Isolated Parallel Plate Capacitor

(5 pts) Use a large grid for this one. Fix the edges at zero. Designate two parallel rows of numbers in the middle of the grid to represent the capacitor. Make sure there is enough space between them, as well as between them and the edges, that you can see what is going on. Fix one of the number blocks at $V = +10$ and the other at $V = -10$. Does the answer match your expectations?

5 Charged, Isolated vs. Grounded Conductors

(5 pts) Take the same grid as the previous section, but now the plate that's at the negative potential should be set to zero. Does your answer match what you'd expect?

6 Grounded Faraday Cage

(5 pts) Set one side of your grid to a high potential, and the opposite side to zero. The remaining two sides can be interpolated, left at zero, or allowed to float – it won't matter. Pick four points in the interior of the grid that are at the corners of a square, centered on the center of the grid. They should be at least 20 elements apart and at least 30 elements from the edges. Force those points to be zero and find the solution to Laplace's Equation. Now take the four points in the centers of the four sides of that square and force them to be zero too. Then take the eight points that bisect the segments between the eight previous points and force them to be zero. Keep bisecting the segments by adding zero points until the pattern is obvious. What happens to the electric field inside the square?

7 Isolated Faraday Cage

(5 pts) Now force all the points in the square to remain at the same constant, but that constant can "float", as determined by the elements immediately adjacent to the square. To what solution does the system converge? What is the potential inside the square? What is the field inside the square? What can you say about the charge distribution along the square?