Homework #5: Uncertainty and Conservation Laws

1. Ford, E3.1 (p. 67)

We start with Cobalt-60, which has a Z# of 27 and a B# of 60. From beta-decay, the Z# of the nucleus must go up by one (the electron leaves) to 28, while the B# stays the same. So we now have Nickel-60. If Uranium-238 undergoes alpha decay, we follow this interaction:

$$_{92}U^{238} \rightarrow_2 \alpha^4 +_x Y^z, \tag{1}$$

where we need to find x and z, and thus determine Y. Well, conservation of B# means $z + 4 = 238$, so $z = 234$, while conservation of Z# means $x + 2 = 92$, so $x = 90$. Therefore we will be left with Y =Thorium-234.

2. $_{92}\mathrm{U}^{232}$ undergoes radioactive decay as it emits an alpha particle with a kinetic energy = 5.414 MeV.

a) What is the baryon (B) number and Z number of uranium-232? Why does it get those numbers?

The Z# is 92, because it has 92 protons, each of which has a Z# of 1. It has a B# of 232 because it has 134 neutrons, each of which has a B# of 1, plus 92 from the protons.

b) Using the various conservation laws, determine the Z number and B number of the daughter nuclide. (Make sure you use conservation laws!)

To use the conservation laws, set up a "before" and "after" chart. Before, you have a Uranium nucleus with a Z# of 92 and a B# of 232. After, you have an unknown daughter nucleus and an alpha particle (Z=2,B=4). Z# must therefore drop down to 90, while the B# drops by 4 to get Thorium-228.

3. You are a store manager for a Target megastore. You get an average number of 12735 customers coming into your store on a typical day. Advertising salespeople are always calling you, claiming that their techniques will increase the number of people coming into your store. One in particular claims that she can increase your number of customers in a single day. How many customers would have to come in that day before you could even start to believe it wasn't just a random fluke?

When counting random events, $\sigma = \sqrt{N}$, so we would expect the number of customers on random days to have a standard deviation of about 113, or 100 (to one sig fig). I would need at least 3σ to believe it's not random, and I would prefer 5 or 6. So I would want to see at least 13100 customers, and preferably more like 13300 before I would believe we didn't just have a good, but normal, day.

Remember, we always round uncertainties to a single digit, and we don't give numbers to higher precision than our uncertainties.

4. Invent a conservation law for money. Show how your law applies to the four steps I outlined in class. That is, what is your system? What is the interaction you are considering? Does money change form? What do you need to consider when calculating whether or not money is conserved? (Note that I mean "money", not "wealth" or "value".)

I define my system to be the customer, the cashier, and the till. The interaction is a purchase. Money does change form (bills and coins), but the total number should stay the same. I need to consider how much money is in the till and the customer's pockets before and after the interaction. If money is dropped, or there is a hole in the customer's pocket, or the cashier steals some from the till, the "if" of my law would be violated. To really illustrate this conservation law, you need to make a "before" and "after" chart and show that although some numbers go up and others go down, the totals stay the same.

Some of you tried to overcomplicate this by making your system something huge and vague like "the economy". The system is the stuff that is interacting. That's it. Everything else that is not directly relevant is not included. You need to think like a physicist and not like an economist.

5. You have measured five angles to $\pm 1°$. These angles are 5°, 12°, 68°, 124°, and 180°. How would you write the sine of these angles, with uncertainty? Hint: Pay close attention to units here. $\sin(\theta)$ is unitless, so you don't want your uncertainty in sine to have a unit! Use radians for your uncertainty; that has no units.

Luckily, the uncertainty is the same for all these: $1° = 0.0175 = 0.02$ radians. I use the formula that says

$$\sigma_s = \sigma_\theta \frac{\partial}{\partial \theta} \sin\theta = \cos\theta \sigma_\theta, \tag{2}$$

because the derivative of the sine is the cosine. So, using my trusty calculator, I get $\sin(5 \pm 1) = 0.09 \pm 0.02$, $\sin(12 \pm 1) = 0.21 \pm 0.02$, $\sin(68 \pm 1) = 0.927 \pm 0.007$, $\sin(124 \pm 1) = 0.83 \pm 0.01$, $\sin(180 \pm 1) = 0.00 \pm 0.02$. Think about the shape of the sine wave. Where it is changing rapidly (small numbers, numbers near 180°), you get larger uncertainties, and where it is changing slowly (angles near 90°) you get smaller uncertainties. You could have also (although it would have been more work than this way) found the sine of 4° and 6° (for example) and split the difference. Because of the way the sine curve changes, this would give you the same answers as above.

Note that even though the cosine function goes negative for angles between 90° and 270°, the uncertainty is still positive because both sides of Equation 2 are technically squared. I have taken the square root of both sides in what I wrote down. This gets rid of any negative signs in the cosine function. Also note that I have rounded the uncertainties to a single digit, and then rounded the values of the sines to the same precision.

6. If Arnold were to fire a 10^0 g paper clip at a speed close to the speed of light, what would happen to him? (Assume Arnold has a mass of 10^2 kg. Give your answer to zero sig figs. If you were to account for relativistic effects, the answer would be much larger!)

To do this one properly, you need to construct a before and after chart. The key conserved quantity here is momentum. Before the paper clip is fired, the momentum of the system (the system is Arnold, including the gun and the paper clip) is zero, because nothing is moving ($v = 0$ for all parts of the system). After the gun is fired, the paper clip has a momentum mv in a particular direction. The total momentum must still be zero, so $m_a v_a + m_p v_p = 0$, which means that $m_a v_a = -m_p v_p \rightarrow v_a = -v_p m_p / m_a$.

That is, the ratio of the speeds will be inverse to the ratio of the masses. So since Arnold's mass is 10^5 times bigger than the paper clip, he will be moving one-hundred-thousand times more slowly. But since the paper clip is moving at $\sim 10^8$ m/s, Arnold will be moving at $\sim 10^3$ m/s the other way! (And like I said, if you take relativity into account, it's actually much faster.)

Although certainly energy is conserved, that conservation law isn't really useful to answer this particular question. Before the interaction, you have rest energy in both the paper clip and Arnold. That rest energy is still there after the gun is fired. We have to add kinetic energy to the "after" column, because the paper clip is moving, but it would be wrong to say that Arnold has the same kinetic energy as the paper clip, from energy conservation arguments. Kinetic energy is always positive, so adding the paper clips KE to Arnold would NOT bring the total KE back down to zero. Even if it could *kinetic* energy is NOT conserved, in general, so there's no reason to think it should balance. Whatever the gun does to fire the paper clip must provide enough energy for the kinetic energy of *both* Arnold and the paper clip. The rest energy of the paper clip is NOT being converted into kinetic energy.

7. a) You measure the length of a table to be 2.45 ± 0.06 m and the width to be 0.87 ± 0.02 m. What is the area of the table in square centimeters?

For multiplication, fractional uncertainties add in quadrature, so

$$\frac{\sigma_a}{a} = \sqrt{\frac{\sigma_w^2}{w^2} + \frac{\sigma_\ell^2}{\ell^2}}. \tag{3}$$

Plugging in the numbers, $a = 2.1315$ m^2, and $\sigma_a = 0.0716$ m^2. Converting to square centimeters, $a = (2.1315 \pm 0.0716) \times 10^4$ cm^2. Rounding off properly, $a = (2.13 \pm 0.07) \times 10^4$ cm^2.

b) You measure the length of three (presumed identical) tables set end to end to be 5.96 ± 0.03 m. How long is one table?

The length of one table should be one-third the length of the total, or 1.9866667 m. The uncertainty is also reduced by 1/3, giving a total of 1.99 ± 0.01 m. Note that the uncertainty is smaller, because you are essentially taking an average, and the more samples go into an average (speaking *very* generally), the smaller the uncertainty gets.

c) You measure the diameter of a cylinder to be 1.3 ± 0.2 cm and the length to be 5.2 ± 0.6 cm. You measure the mass to be 4.56 ± 0.01 g. What is the average density of the cylinder?

This is multiplication and division, so we use the "fractional uncertainties add in quadrature" rule. If the density is given by

$$\rho = \frac{m}{V} = \frac{4m}{\pi d^2 \ell}, \tag{4}$$

then the uncertainty in the density is

$$\left(\frac{\sigma_\rho}{\rho}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{2\sigma_d}{d}\right)^2 + \left(\frac{\sigma_\ell}{\ell}\right)^2. \tag{5}$$

The σ_d is multiplied by 2 because it is squared, so when you take the derivative, the 2 comes down and becomes a multiplicative factor. This is to account for the fact that a change in d becomes a *bigger* change in ρ, because d^2 rises faster than d. We know all these numbers, so $\rho = 660.6705$ kg/m^3. The fractional uncertainty, according to Equation 5, is 0.328622, which means that the uncertainty in ρ is 217 kg/m^3. Rounding things off properly means $\rho = 700 \pm 200$ kg/m^3.

There is no point in calculating what the volume actually is. It's a lot of unnecessary work, and it violates the dictum that you should "always plug in numbers last." In fact, it's actually a bad idea to do so, because each step you take in processing the numbers increases your uncertainty (also, you get rounding errors, too).

d) You measure the length of a table to be 1.98 ± 0.03 m. How long would three of these tables placed end to end be?

We're adding now, so the uncertainties add in quadrature. There are three tables, so the uncertainty in all three tables together will be $\sqrt{3}$ times the uncertainty in a single table. So the answer is 5.94 ± 0.05196 m. Rounding off properly yields 5.94 ± 0.05 m. Alternately, you could multiply the length of one table by three, and then your answer would be 5.94 ± 0.09 m. Note that while this answer is not different from the length of the three tables in part (b), the uncertainty is larger. Also note that the "correct" uncertainty depends on what you *do* (it's an operational definition)! Adding three numbers together is not the same thing as multiplying by three, when you're dealing with uncertainties. (You will remember something similar when you found the volume of a sphere: d^3 yielded a different uncertainty than $d \times d \times d$.)