Homework #3: Units and Uncertainty

1. Approximately how long would a typical Hollywood movie be if you took it off the projector and unrolled it out to a straight line?

I know that a movie typically lasts about 90 minutes, and that the film shows 24 frames every second. That makes 129,600 frames in a typical movie. Standard film stock is 35 mm wide, and has an aspect ratio of 4:3, so it would be 26.25 mm tall. I don't know how big the space between frames is, but I am going to assume it's something like three mm, which would enable me to assume each frame is 30 mm tall. That's 0.03 m per frame, multiplied by the number of frames yields 3888 m, or 4 km, to one sig fig.

2. **Setup:** I am asked to invent an operational definition for an everyday concept. An operational definition is a procedure that leads to a measurement, such that anyone, anywhere can perform the procedure and agree on the outcome. **Work and Answer:** My favorite example that one of you thought up was a definition of "Homework Crunch Index," in which you count the number of problems remaining, divided by the remaining time until the homework is due. Thus, as the due date looms, this number gets bigger and bigger, although actually doing the homework can reduce it. This person also suggested that the SI unit for this quantity be the "cram". I suppose 100 crams would be a "prayer." **Comment:** This is a clear, unambiguous, procedural definition that results in a specific measurement that everyone would agree on.

3. In this problem, we are trying to use dimensions to estimate physical quanities. If you throw a ball or stone with a speed v, and it is subject to a constant gravitational acceleration of g, then we can combine v and g in ways to get dimensions of length and time. v^2/g gives length, and v/g gives time. It is important to note that v is the *initial* speed, not the *average* speed. That is, $v \neq R/t$. A real thrown object will change speed during flight, and so the average speed will not, in general, be equal to the initial speed.

In a real situation, that is, throwing a rock, the length the rock travels and the time it takes to get there will not *exactly* be these quantities, but will be related by some dimensionless constant. We therefore get two equations, one for the range, R, and one for the time t:

$$R = K_1 \, \frac{v^2}{g} \tag{1}$$

$$t = K_2 \, \frac{v}{g} \tag{2}$$

To get an estimate for K_1 and K_2, one can take $g = 10$ m s^{-2}, and then one needs to throw a rock at roughly 45° to the horizontal at a known speed. To estimate the speed, I recommend spinning the rock in a circle (like an underhand softball pitch). My arm is about 1 m long, so if I spin the rock in a circle at a constant speed for one second, it will have travelled about 6 m around the circle, and when I let it go, it will be going about 6 m/s.

To one significant figure, the ball I threw in this manner took about one second to fly about 5 m. So I would estimate K_2, from Equation 2 to be about 1.6, or 2. From Equation 1, I get K_1 to be about 1.4, or 1.

I would be very surprised if these numbers were off by an order of magnitude, and indeed you can calculate that in a world without air resistance, a rock thrown at a perfect 45° angle would yield $K_2 = 0.7$ and $K_1 = 1$.

4. Using Planck's constant (h), the speed of light (c), and the mass of a proton (m_p), construct quantities with the dimensions of (a) length, (b) time, and (c) energy. Then evaluate those quantities in units of (a) fm, (b) sec, and (c) eV.

First of all, let me write down the values of the constants. I will use three significant figures. $c = 3.00 \times 10^8$ m/s (dimension of length divided by time), $h = 6.63 \times 10^{-34}$ Js (dimension of mass times length^2 divided by time), and the mass of the proton is $m_p = 1.67 \times 10^{-27}$ kg (dimension of mass).

If I want dimensions of length, I can h/m_p, which will give me length^2 divided by time. If I then divide by c, one power of length, and the time, will cancel. So $h/(cm_p)$ has dimensions of length. The value in fm would be 1.32 fm.

If I want dimensions of time, I note that m_pc^2 has dimensions of energy ($E = mc^2$, after all), so if I take $h/(c^2m_p)$, the energy dimensions will cancel, leaving me with time. That is, I take the previous length, and divide by c to get time. The value would therefore be 1.32 fm/c, which is 4.41×10^{-24} s.

To get an energy, I use m_pc^2, as mentioned above. This yields 1.50×10^{-10} J, which is 9.38×10^8 eV. This is the rest energy of the proton.

The point of these numbers is to notice that they are all very small. That is, although c is large, h and m_p are so small that the results are also tiny. This means, whatever these lengths and times might be, they are *very* sub-atomic.

5. In the lab, you measure four quantities: $a = 5 \pm 1$ cm, $b = 18 \pm 2$ cm, $c = 12.2 \pm 0.5$ cm and $m = 18 \pm 1$ g. Compute the following quantities. This problem is practice in propogating uncertainties. For a) and b), I will add the uncertainties in quadrature. For part d), I will add the fractional uncertainties in quadrature.

a) $f = a + b + c$. $\sigma_f = \sqrt{\sigma_a^2 + \sigma_b^2 + \sigma_c^2} = \sqrt{1 + 4 + 0.25} = 2.29$ cm. Therefore, $f = 35 \pm 2$ cm.

b) $f = a + b - c$. The uncertainty here is the same as in part a). Here, I just have a different sum. $f = 11 \pm 2$ cm.

c) $f = a - b + m$. This can't be done. You can't add mass to length. Addition must have the same dimensions.

d) $f = ma/b$. Here, I use fractional uncertainties:

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 = \left(\frac{1}{18}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{2}{18}\right)^2 = 0.0554. \quad (3)$$

This means, since $f = 5$ g, that I should write my final answer as $f = 5 \pm 1$ g. That's about a 20% uncertainty, which is not smaller than any of the fractional uncertainties that went into it (6%, 11% and 20%). Clearly, the uncertainty in a dominates over the others.

6. You measure the diameter of a sphere to be $(3.25 \pm 0.03) \times 10^{-2}$ m. What is the volume of the sphere? We know the volume to follow the formula $V = 4\pi r^3/3$, which in terms of the diameter, d, is $V = \pi d^3/6$. Plugging the measured value for d in, I get $V = 1.7974 \times 10^{-5}$ m^3. The uncertainty, on the other hand, comes from the derivative formula for propogating uncertainties:

$$\sigma_V^2 = \left(\frac{\partial V}{\partial d}\sigma_d\right)^2 \rightarrow \sigma_V = 3 \times \frac{\pi}{6} d^2 \sigma_d, \quad (4)$$

or

$$\frac{\sigma_V}{V} = 3\frac{\sigma_d}{d} \quad (5)$$

Therefore, the fractional uncertainty in the volume should be 3 times the fractional uncertainty in the diameter, or $\sigma_V/V = 3\ \sigma_d/d = 2.76923 \times 10^{-2} \rightarrow \sigma_V = 4.9774 \times 10^{-7}\ \mathrm{m}^3$. Putting the two together, I get $V = (1.7974 \pm 0.04977) \times 10^{-5}\ \mathrm{m}^3$, and rounding off to the correct precision, I get $V = (1.80 \pm 0.05) \times 10^{-5}\ \mathrm{m}^3$. That's a little less than three percent uncertainty, and the diameter was measured to better than 1%, so the uncertainty went up by a factor of three, which is what we would expect, since volume goes up by the cube of the radius.

7. Look at the data you collected for your first lab. How small would a number in a given bin have to be before you should be suspicious that something weird was going on in that interval (like someone putting their hand in front of the detector)?

If you had approximately 100 counts per 5-s interval, then I imagine your 68% uncertainty range was something like 90 to 110. In that case, I wouldn't believe a single low interval out of 100 samples had something wrong with it unless it was below 70 *at most*, and I'd be more comfortable saying something was wrong with it if it were around 50 or 40 or less. Three sigma is really the limit at which you can start to believe there's more than just randomness going on (with 100 samples).

To put it another way, with 100 samples, you would *expect* about one sample to yield a result outside the 99% range. If you want to believe that one sample out of 100 samples is significantly outside the distribution, the odds of getting it had better be smaller than 1%. So I would go with at least 4σ, and 5 or 6σ would be even better.

8. A pollster reports that candidate Smith has 49% of the vote, while candidate Jones has 48% of the vote. How small would the uncertainty in those numbers have to be (assume each has the same uncertainty) before you even *start* to think that candidate Smith might be winning?

Again, three sigma is the limit, so you want the difference between them to be at least three sigma. I would demand at most 0.3% or 0.2% uncertainty before I would start to believe that those two numbers were different.